THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MID SESSION TEST - MAY 1997

PHYS2001 - MECHANICS AND COMPUTATIONAL PHYSICS

PHYS2999 - MECHANICS AND THERMAL PHYSICS

PAPER 1 - MECHANICS

Time allowed - 50 minutes

Total number of questions - 2

This paper may be retained by the candidate

P.T.O.

The following equations are supplied as an aid to memory.

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

Damped Harmonic Motion

If $m\ddot{x} + c\dot{x} + kx = 0$

then $x = Ae^{qt}$

where $q = -\gamma \pm (\gamma^2 - \omega_o{}^2)^{1/2}$

$\gamma = c / 2m$

$\omega_o{}^2 = k / m$

$\omega_d{}^2 = \omega_o{}^2 - \gamma^2$

Forced Harmonic Motion

If $m\ddot{x} + c\dot{x} + kx = F_o \cos \omega t$

then $x = A \cos(\omega t - \phi)$

where $$A = \frac{F_o}{\left[m^2\left(\omega_o{}^2 - \omega^2\right)^2 + c^2\omega^2\right]^{1/2}} = \frac{F_o}{m\left[\left(\omega_o{}^2 - \omega^2\right)^2 + 4\gamma^2\omega^2\right]^{1/2}}$$

and $$\tan \phi = \frac{c\omega}{m\left(\omega_o{}^2 - \omega^2\right)} = \frac{2\gamma\omega}{\left(\omega_o{}^2 - \omega^2\right)}$$

resonance $\omega_r{}^2 = \omega_o{}^2 - 2\gamma^2$

$$Q = \frac{\omega_d}{2\gamma}$$

Central Forces

Polar Coords

$\mathbf{r} = (r, \theta)$

$\mathbf{v} = (\dot{r}, r\dot{\theta})$

$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2, 2\dot{r}\dot{\theta} + r\ddot{\theta})$

$$u = \frac{1}{r} \rightarrow \frac{d^2u}{d\theta^2} + u = -\frac{1}{mh^2u^2} f(u^{-1})$$

$$e = \left(r_a - r_p\right) / \left(r_a + r_p\right)$$

$$e = \frac{mh^2}{kr_p} - 1$$

h = angular momentum / unit mass

Apsidal angle $$\psi = \pi\left(3 + a\frac{f'(a)}{f(a)}\right)^{-1/2}$$

Stability $$f(a) + \frac{a}{3}f'(a) < 0$$

Inverse Square Law Orbits

$$V = -\frac{k}{r}$$

Gravitation $$k = GMm$$

$$\dot{\theta} = hu^2$$

$$f(r) = -k/r = -\frac{GMm}{r^2}$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

$$e = (r_a - r_p)/(r_a + r_p)$$

$$e = \frac{mh^2}{kr_p} - 1$$

$$\tau = \frac{2\pi}{\sqrt{GM}}a^{3/2}$$

$$G = 6.67\times10^{-11}\ N\ m^2kg^{-2}$$

$$M_{sun} = 2\times10^{30}\ kg$$

$$M_{earth} = 6\times10^{24}\ kg$$

$$R_{earth} = 6400\ km$$

Lagrange's Equations

$$L = T - V$$

$$\frac{d}{dt}\frac{\partial L}{2\dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

Generalized momenta $$p_j = \frac{\partial L}{\partial \dot{q}_j}$$

$$H = \sum p_i\dot{q}_i - L$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i},\quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

QUESTION 1

A particle moves in one dimension under the action of a conservative force.

(a) Show that the formal solution to the equation of motion is given by:

$$t - t_o = \int_{x_o}^{x} \frac{dx}{\sqrt{\left(\frac{2}{m}\right)\left(E - V(x)\right)}}$$

where the particle is at x_o at time t_o.

A central force

$$\underline{F} = f(r)\,\hat{\underline{r}}$$

is conservative.

(b) Show that the central force problem can be reduced to a purely radial problem with the same formal solution as in part (a) except that the potential V(r) is replaced by an "effective potential".

(c) Describe the nature of the "effective potential" and from where the extra potential terms arise.

(d) Derive a formal solution for the orbit of a particle in a central field using the solution of part (a) as your starting point (i.e. express θ in terms of r).

QUESTION 2

Part A

For the damped harmonic oscillator, the equation of motion can be written as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(a) explain all terms in this equation.

The equation can be solved by finding the roots of the corresponding subsidiary equation:

$$m\alpha^2 + c\alpha + k = 0$$

This results in three distinct solutions that represent three types of physical behaviour: overdamping, critical damping and underdamping.

(b) Explain each of these modes of physical behaviour. Use sketches to illustrate the motion of the particle in each case.

Part B

A new physical system is designed having the following equation of motion:

$$m\ddot{x} + c\dot{x} - kx = 0$$

where m is the mass of a particle and c and k are constants.

(c) From the corresponding subsidiary equation (or otherwise) find the solution to this equation of motion (Hint: use the solution of the damped harmonic oscillator as a guide).

(d) How many distinct types of solution and hence physical behaviour, does this system exhibit? (i.e. does it have solutions that correspond to "overdamping, critical damping and underdamping"?)

(e) Discuss the physical reasons for the difference in the physical behaviour of this system compared with the damped harmonic oscillator.