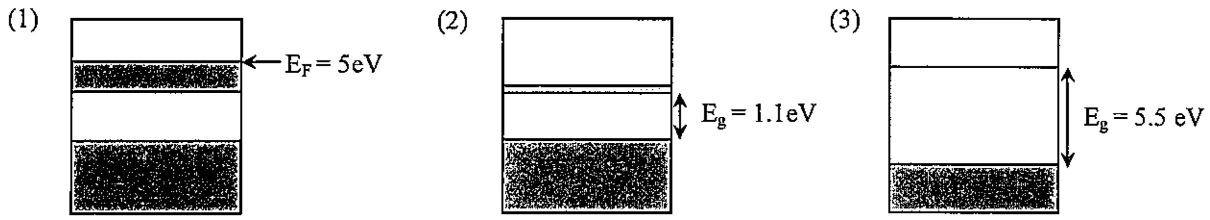
PHYS 1231

QUESTION 4 **[Marks 10]**

(a) Three materials have the energy band structures shown schematically in the diagram below representing, (1) a metal, (2) an n-type doped semiconductor and (3) an insulator. The shaded aťeas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity and the thermal velocity of the electrons at 300K.

(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2).

(iii) By comparing the energy gap values for materials (2) and (3) state, with your reasoning, whether material (3) is expected to be transparent or opaque to visible light at room temperature. (The visible region of the EM spectrum spans the wavelength range λ=400nm to λ=700nm approx.)

(b) Copper (Cu) metal is monovalent (1 conduction electron per atom) and has density $\rho = 8.9x10^3$ kg.m^{-3} and atomic mass 63.5. Use this information to estimate the number of occupied electron energy levels in the conduction band of 1 mm^3 of copper at 300K.

SOLUTION

(a) (i) Fermi velocity

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2(5.0x10^{-19}J)}{9.1x10^{-31}kg}} \approx 1.1x10^6\ ms^{-1}$$

Thermal velocity

$$v_{th} = \sqrt{\frac{2k_BT}{m_e}}$$

$$= \sqrt{\frac{2(1.38x10^{-23}\ JK^{-1})(300K)}{9.1x10^{-31}}}$$

$$\approx 1x10^5 ms^{-1}$$

(ii) $$\lambda_g = \frac{\mathrm{hc}}{\mathrm{E_g}} = \frac{(6.63x10^{-34})(3x10^8)}{(1.1)(1.6x10^{-19})} = 1.1x10^{-8}\mathrm{m}$$

(iii) Material (2) is opaque because the band gap magnitude permits absorption of all wavelengths in the visible spectrum (400nm-700nm). Material (3) is transparent because the band gap magnitude means visible wavelengths are not absorbed.

(b) Copper has number of carrier per m^3 is n given by $n = \frac{\rho N_A}{M}$ where ρ is the density, N_A is Avogadro's number, M is the atomic mass.

$$n = \frac{\rho N_A}{M} = \frac{(8.9x10^3)(6.023x10^{26})}{63.5} = 8.44x10^{28}\ m^{-3}$$

since all available states in the conduction band are occupied n is also the number of occupied states

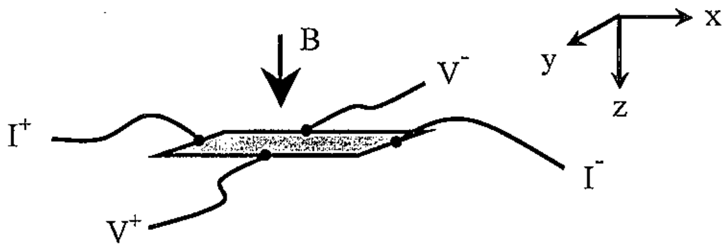
QUESTION 5 **[Marks 14]**

(a) The semiconductor indium antimonide (InSb) has energy gap $E_g = 0.23$ eV. The dielectric constant of InSb is $\varepsilon = 17$ and the electron effective mass is $m_e^* = 0.014m_e$. Assuming a simple Bohr model calculate,

(i) the donor ionisation energy,

(ii) the radius of the ground state donor electron orbit.

(b) A Hall effect probe consisting of a thin, rectangular slice of silicon doped at a concentration $n = 1.0\text{x}10^{20}\text{m}^{-3}$ is used to measure the total stray magnetic field above an MRI imaging machine. The Hall probe is positioned so that the magnetic induction B is perpendicular to the face and along the thin direction (z-direction) of the slice, as shown below. The slice has dimensions length l (parallel to x-direction), width w (y-direction) and thickness t (z-direction). A constant current of 1.0 mA is passed from I^+ to I^-. The induced Hall voltage, V_H, is measured on wires labelled V^+ and V^-.



(i) Use the condition for equilibrium of the magnetic and electric forces on the charge carriers, and the free electron formulae given at the front of this paper to derive an expression which gives the Hall voltage in terms of the magnetic induction B_Z, the current I_X, the carrier concentration n, charge e and the sample dimensions.

(ii) When the silicon slice carries a constant current 0.1 mA (I^+ to I^-) a Hall voltage of magnitude 2.0 mV is measured. Calculate the value of the stray magnetic field B.

(iii) When the conventional current I^+ to I^- is in the +ve x-direction it is found that probe V^- is positive (with respect to V^+). Is the silicon slice p-type doped or n-type doped? You must provide your reasoning, a simple statement of the answer, p-type or n-type without a supporting argument will not receive marks. Your argument could be a series of dot points connected with "therefore" or a short paragraph of complete sentences.

SOLUTION

(a) (i) $E_d = -\frac{13.6}{\varepsilon_r^2}\left(\frac{m_e^*}{m_e}\right) \text{ eV} = -\frac{13.6}{17^2}(0.014) = 6.6\text{x}10^{-4} \text{ eV}$

(ii) $a_d = a_0\varepsilon_r\left(\frac{m_e}{m_e^*}\right) = 0.053(17)\left(\frac{1}{0.014}\right) = 64 \text{ nm}$

(b)
(i) When the Hall voltage is established the force on the electrons is balanced, so that

$$qE_y = B_z q v_x$$

The electric field in the y-direction is the Hall field caused by the Hall effect; that is

$$E_y = E_H = \text{Hall electric field}$$

Using $I = nAve$ (Drude formula) the current density J is

$$J_x = \frac{I_x}{A} = nqv_x$$

The subscripts on J, I and v give their directions. Since $J = \sigma E$ where σ is the conductivity, we have

$$J_x = \frac{I}{A} = \sigma E_x = nqv_x$$

Rearranging these we find that

$$E_y = \frac{B_z J_x}{nq} = \frac{B_z E_x \sigma}{nq}$$

Note that

$$E_y = \frac{V_y}{w} = \frac{V_H}{w}$$

where V_H is the Hall voltage, w is the width of the semiconductor specimen in metres.
If the current, I, and magnetic field, B, are known, measurement of the Hall voltage V_H gives us the electron concentration *n*:

$$n = \frac{B_z J_x}{E_y q} = \frac{w B_z J_x}{V_H q}$$

or

$$n = \frac{w B_z I_x}{A V_H q}$$

where A is the cross-sectional area of the specimen.
Since A=w x t (cross-sectional area A = width w times thickness t) we can also write,

$$n = \frac{BI}{t V_H q}$$

or

$$V_H = \frac{BI}{nqt}$$

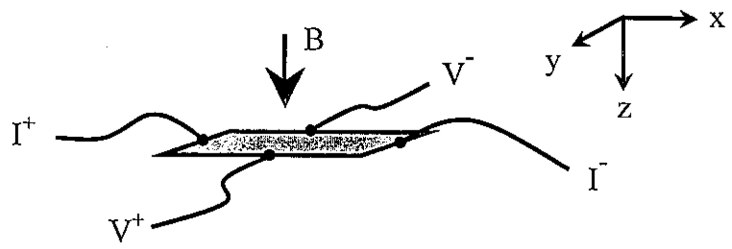
where B is the magnetic induction (tesla), I is the current (amps) through the specimen (or Hall probe), q is the carrier charge ($= -e = -1.6\text{x}10^{-19}$ C for electrons and $= +e = +1.6\text{x}10^{-19}$ C for holes) and t is the specimen thickness.

(ii) From $V_H = \dfrac{BI}{nqt}$ the stray magnetic field is

$$B = \frac{V_H nqt}{I} = \frac{(2.0\text{x}10^{-3})(1.0\text{x}10^{20})(1.6\text{x}10^{-19})}{0.1\text{x}10^{-3}} t = (320).t \text{ tesla}$$

(a typical value of t is 0.1-0.5mm giving a stray field less than 1 T)

(iii) When the conventional current I^+ to I^- is in the +ve x-direction it is found that probe V^- is positive (with respect to V^+). Is the silicon slice p-type doped or n-type doped?



- Electron drift velocity is in the –x direction.
- Magnetic force on charge carriers is $F_m = q(\mathbf{vxB})$ in the –y direction
- F_m causes –ve charge to accumulate on the V^- probe. This accumulation of charge would mean a Hall voltage measured positive from V+ to V- but the measured voltage is the other way around
- Therefore, the charge carriers are holes and the doping is p-type.

QUESTION 6 **[Marks 16]**

(a) The current-voltage characteristic for an ideal diode in the forward direction is given by $I = I_0\left(e^{e\Delta V/k_B T} - 1\right)$

(i) Give the meaning of the symbols in this expression.

(ii) Provide a labeled sketch graph of the form of the current-voltage characteristic expected for an ideal diode.

(iii) Sketch a simple circuit diagram showing a forward-biased diode. Your *series* circuit should contain the diode, a battery and a single resistor R only.

(iv) The diode in the circuit of part (iii) is rated at 100mA maximum current. If the battery voltage is $\varepsilon = 1.5\ \mathrm{V}$, calculate the minimum value of R required.

(b) The IR communication port on your PDA (e.g. like the Palm Pilot or pocket PC) operates with an infra-red LED. *Estimate* the band gap of the semiconductor required for the LED in this device.

(c) Light of wavelength $\lambda = 620\ \mathrm{nm}$ $(1\mathrm{nm} = 10^{-9}\mathrm{m})$ and intensity $2.0\ \mathrm{Wm^{-2}}$ is shone onto the surface of a photoconductive detector consisting of a rectangular specimen of semiconductor. The detector has an effective area $1.0\mathrm{x}100.0\ \mathrm{mm}^2$. The mobilities μ of the electrons and holes in the semiconductor are $0.001\ \mathrm{m^2V^{-1}s^{-1}}$ and $1.0\ \mathrm{m^2V^{-1}s^{-1}}$ respectively. Assuming each photon of light striking the semiconductor is absorbed and produces an electron-hole pair, calculate the change in electrical conductance produced by the illumination.

[Hint: the change in conductance is $\Delta g = \Delta\sigma\frac{A}{L}$. The conductivity is $\sigma = ne\mu$ where $n = \frac{\Delta N + \Delta P}{AL}$ for ΔN photoexcited electrons and ΔP photoexcited holes and $AL = V$ is the volume of the semiconductor detector.]

SOLUTION

(a) (i) In the equation $I = I_0\left(e^{e\Delta V/k_B T} - 1\right)$ the symbols have the meaning

I_0 is the saturation reverse bias current

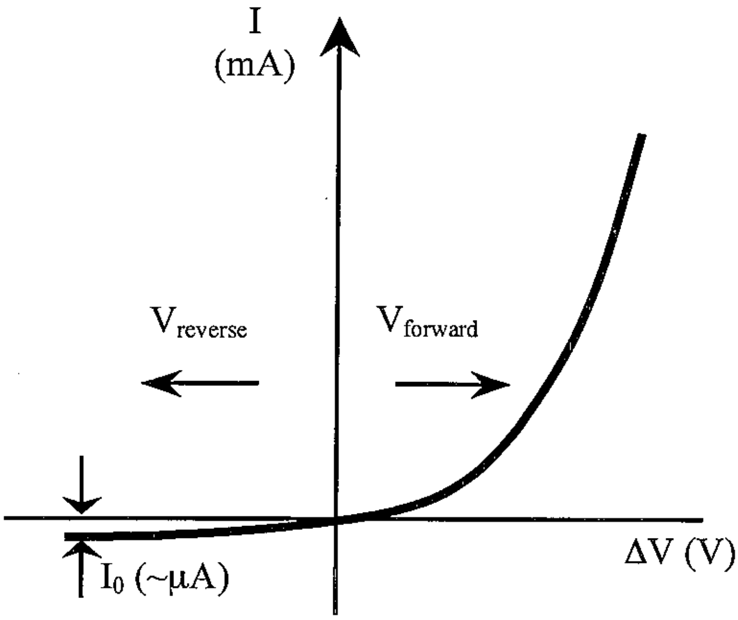
e (in exponent) is the electron charge

ΔV is the diode bias voltage

k_B is Boltzmann's constant

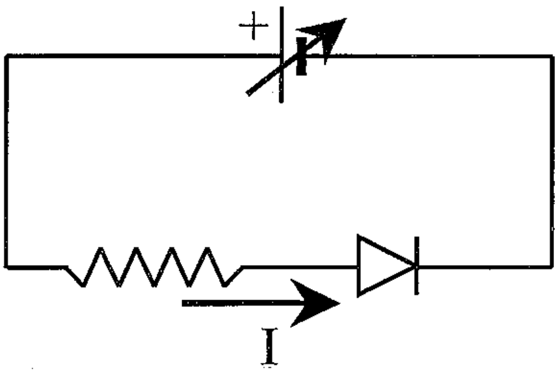
T is the temperature in Kelvin

(ii) The ideal diode has I-V characteristic :



(iii) Circuit for forward biased diode:

Marker Note: battery polarity must be correct for forward bias (as shown) to obtain marks



(iv) $R = \dfrac{(1.5 - 0.6)}{100\text{x}10^{-3}} = 9\ \Omega$

(b) Infra-red LED, take $\lambda = 900\text{nm}$.

(**Markers Note:** N.B. this is for an estimate only. Other IR wavelengths are also acceptable.)

$$E = \frac{hc}{\lambda} = \frac{(6.63\text{x}10^{-34})(3\text{x}10^{8})}{900\text{x}10^{-9}} = 2.2\text{x}10^{-19}\ \text{J} \approx 1.4\ \text{eV}$$

(c) The photon energy is

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.63\text{x}10^{-34})(3\text{x}10^{8})}{620\text{nm}} = 3.2\text{x}10^{-19}\ \text{J}$$

The number of photons per second striking the semiconductor is n_{photon}

$$n_{photon} = \frac{P}{E_{photon}} = \frac{2.0}{3.2x10^{-19}}\left(10^{-4}\ m^2\right) = 6.25x10^{14}$$

(the effective surface area S of the photodetector is $1mmx100mm = 10^{-4}\ m^2$)

There are $6.25x10^{14}$ e-h pairs created assuming each absorbed photon creates a pair.

$\Delta N, \Delta P$ are the number of extra (photo-excited) electrons and holes due to the illumination.

The conductivity is $\sigma = ne\mu$. The *conductance* is $G = \sigma\frac{A}{L}$. (A is the cross-sectional area of photodetector, L is length of photodetector)

Using $\sigma = ne\mu$, in the presence of illumination,

$$\Delta\sigma = \frac{e(\Delta N\mu_e + \Delta P\mu_h)}{AL}$$

where μ_e, μ_h are the respective electron and hole mobilities and AL is the volume. The change in conductance is

$$\Delta G = \frac{e(\Delta N\mu_e + \Delta P\mu_h)}{L^2}$$

giving

$$\Delta G = \frac{(1.6x10^{-19}C)\left[(6.25x10^{14})\mu_{el} + (6.25x10^{14})\mu_h\right]}{L^2}$$

substituting the electron and hole mobilities and putting $L = 100\ mm = 100x10^{-3}m$,

$$\Delta G = \frac{(1.6x10^{-19}C)\left[(6.25x10^{14})(1.0) + (6.25x10^{14})(0.001)\right]}{(100x10^{-3})^2} = \frac{10^{-4}}{10^{-2}}$$

$$= 0.01\ S \quad \text{(siemens)}$$

Marker Notes:

(i) due to much lower mobility the hole contribution to the conductance is assumed negligible

(ii) students may take the length of the photodetector as 1mm which gives $\Delta G = \frac{10^{-4}}{10^{-6}} = 10^2\ S$. Either answer gains full marks.