Solutions

PHYS ~~[illegible]~~ 1221R

Question 7 (18 marks)

a) Assuming that $F = -kx$ for the elastic cords, the seat (and baby) execute vertical SHM with an amplitude of 10 cm about an equilibrium position of −18 cm } 6

b) $f = \frac{1}{2\pi}\sqrt{k/m}$

$k = -F/x = \dfrac{12\,\text{kg} \times 9.8\,\text{ms}^{-2}}{0.18\,\text{m}}$

$= 653\ \text{N/m}$

$\therefore f = \frac{1}{2\pi}\sqrt{\dfrac{653\,\text{N/m}}{(12+5)\,\text{kg}}}$

$= 0.99\ \text{Hz}$ } 6

c) $y = y_{max}\cos\omega t$

$\therefore v = dy/dt = -y_{max}\,\omega \sin\omega t$

$\therefore v_{max} = -y_{max}\,\omega = 0.18 \times 2\pi \times 0.99$

$= 1.12\ \text{m/s}$ 4

This occurs as the baby passes in either direction through the equilibrium position of −18 cm 2

~~d) $a = dv/dt = -y_{max}\,\omega^2 \cos\omega t$~~

~~Baby + bouncer part company when $a = g$~~

~~$\therefore y_{max} = g/\omega^2 = \dfrac{9.8}{(0.99 \times 2\pi)^2} = 0.25\ \text{m}$~~

Question 8 (18 marks)

a) Aluminium $l_1 = \frac{n_1}{2} \cdot \lambda_1$

$= \frac{n_1}{2} \cdot \frac{v_1}{f}$ 3

But $v_1 = \sqrt{T/\mu} = \sqrt{\frac{mg}{\rho_1 A}}$

$\therefore l_1 = \frac{n_1}{2f}\sqrt{\frac{mg}{\rho_1 A}}$ $\therefore f = \frac{n_1}{2l_1}\sqrt{\frac{mg}{\rho_1 A}}$ 3

Similarly, for the steel:

$f = \frac{n_2}{2l_2}\sqrt{\frac{mg}{\rho_2 A}}$

But f is the same in both cases.

$\therefore \frac{n_1}{2l_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{n_2}{2l_2}\sqrt{\frac{mg}{\rho_2 A}}$ 3

$\therefore \frac{n_2}{n_1} = \frac{l_2}{l_1}\sqrt{\frac{\rho_2}{\rho_1}} = 2.5$ 2

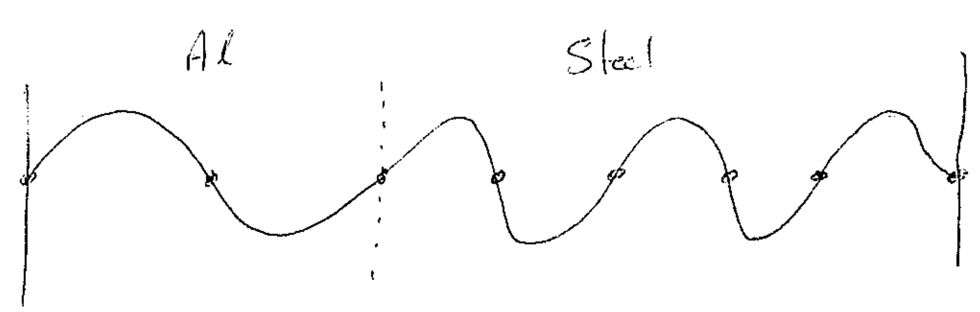
Smallest integer values are therefore

$n_2 = 5$, $n_1 = 2$ 1

$\therefore f = \frac{n_1}{2l_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2 \times 0.60}\sqrt{\frac{10 \times 9.8}{2.60 \times 10^3 \times 1.00 \times 10^{-6}}}$

$= 323$ Hz 3

b)



= 6 nodes 3

Question 4 (18 marks)

a) i) Car and observer have zero relative motion along line-of-sight.

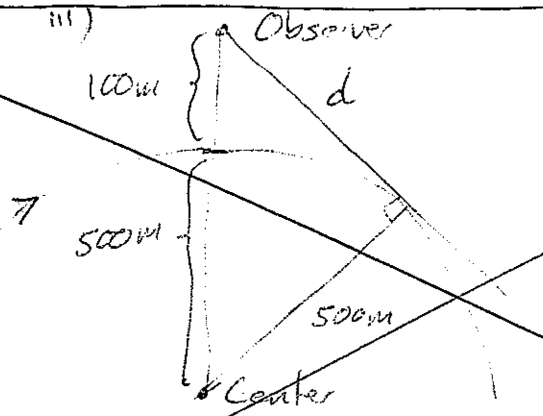
$\therefore f = 12 \text{ kHz}$ 5

ii) $f = f_o \left(\frac{V + V_o}{V + V_s} \right)$

$V_s = 250/3.6 = 69.4 \text{ m/s}$

$\therefore f = 12.0 \left(\frac{343}{343 + 69} \right) = 9.98 \text{ kHz}$ 6

~~iii)~~



~~Car is receding from observer along a tangential line.~~ ~~$d = \sqrt{600^2 - 500^2}$~~

~~$= 332 \text{ m}$ away~~

Note: Better students might take the transit time of the sound into account. So, in part i) when the car is directly opposite the observer the sound heard was actually emitted when the car was approaching the observer and ~20 m before the point of closest approach. Similarly ~~in ii) the car will have moved a further~~ ~~$332 \times \frac{69.4}{343} = 67$ m around the track when the observer~~ hears the lowest note. Such students deserve extra points, if not extra marks.

Question 9

b) $$I = \tfrac{1}{2}\rho v \omega^2 s_{max}^2$$

$$\therefore \quad s_{max} = \sqrt{I \cdot 2 / \rho v \omega^2}$$

$$= \sqrt{\frac{2 \times 10^{-12}}{1.2 \times 343 \times (2\pi \times 4 \times 10^3)^2}}$$

$$= 2.77 \times 10^{-12}\ m \qquad 5$$

which is 100 times smaller than a molecule of air! 2

Question 10 (16 marks)

a) Standard form of the wave equation is

$$E = E_0 \sin(kx - \omega t) \qquad 3$$

Compare $E = 50 \sin \pi (4.50 \times 10^6 x - 9.00 \times 10^{14} t)$ V/m

i) $k = \pi \times 4.50 \times 10^6 \ m^{-1}$ = 3

But $\lambda = 2\pi / k$

$\therefore \lambda = 4.44 \times 10^{-7}$ m

ii) $k = 1.41 \times 10^7 \ m^{-1}$ 3

iii) $f = \omega / 2\pi = 1/2\pi \times 9\pi \times 10^{14} = 4.50 \times 10^{14}$ Hz 3

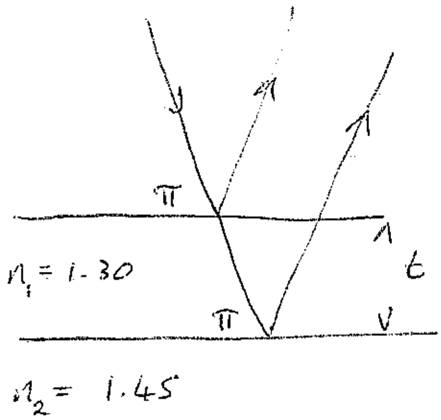
iv) $E = 50$ V/m 3

b) $v = \lambda f = 4.44 \times 10^{-7} \times 4.50 \times 10^{14}$

$= 2.00 \times 10^8$ m/s

$\therefore n = c/v = 1.50$ 3

Question 11 (16 marks)



4

Destructive interference when $2nt = m\lambda$

i.e., $\lambda = \frac{2nt}{m} = \frac{2 \times 1.30 \times 670 \text{ nm}}{m}$ 6

m	= 1	λ	=	1742 nm
	2		=	871 nm
	3		=	581 nm
	4		=	436 nm
	5		=	384 nm

∴ Values between 400 and 750 nm are

436, 581 nm. 6

Question 12 (14 marks)

a) i/ $d \sin\theta = m\lambda$

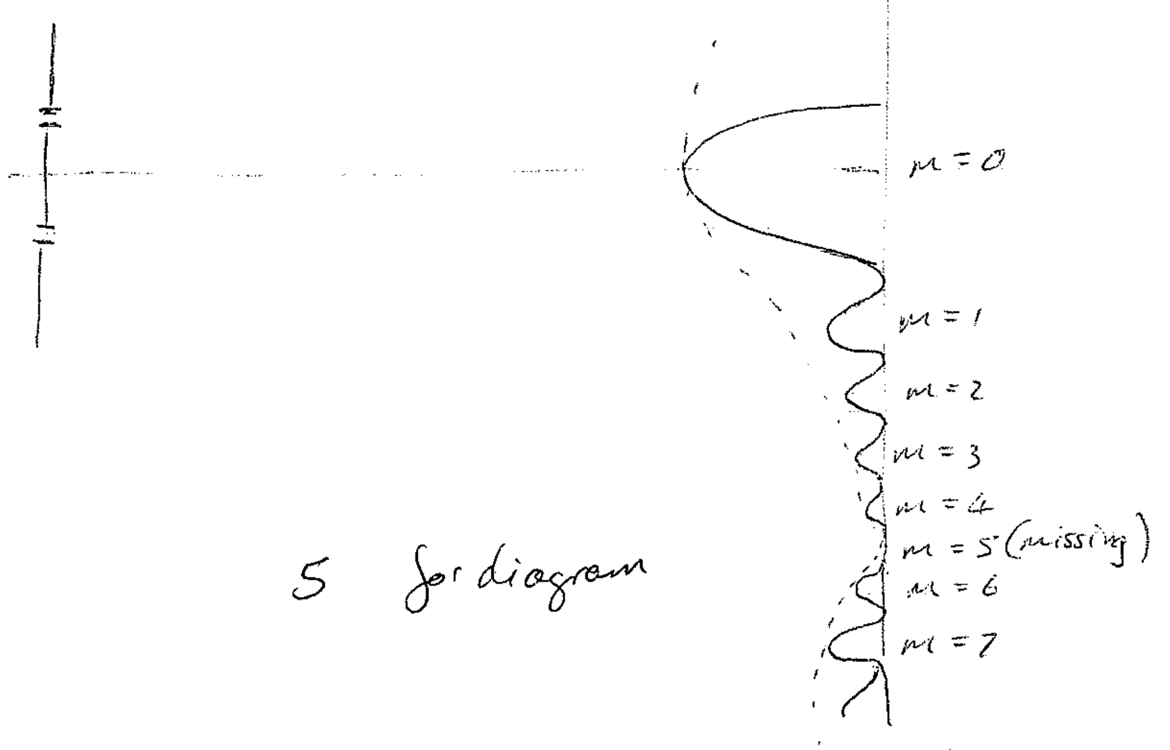
∴ Fringe separation is $L\sin\theta = \frac{L\lambda}{d}$ 3

$$= \frac{1.00 \times 600\times10^{-9}}{0.5\times10^{-3}}$$

$$= 1.20 \text{ mm}$$ 6

ii) Minimum of diffraction pattern at $m = d/a$,

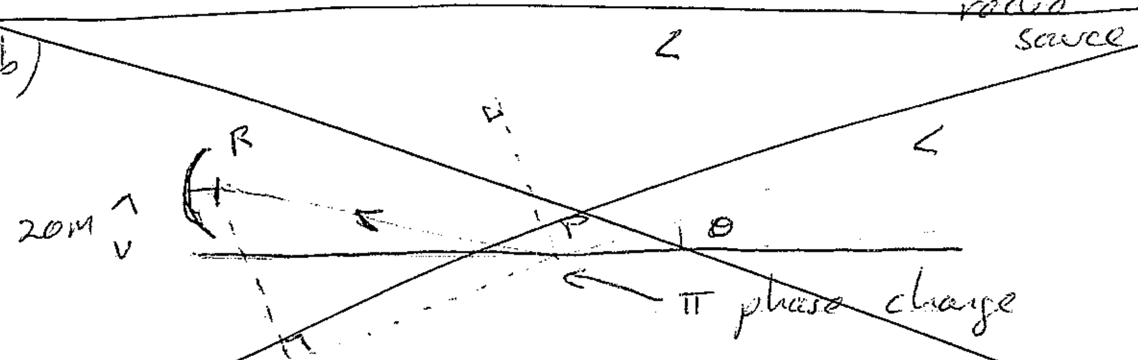
∴ $m = 5$



5 for diagram

~~radio source~~

~~b)~~



~~Phase change of π at sea.~~

~~∴ require path difference of λ/2 for maximum~~