Answer 1

(a)
- $Q' = Q$, the charge remains the same, since there is nowhere for the charge to flow.
- $C' = \kappa C$.
- $V' = V/\kappa$, since $V = Q/C$ and C has increased.
- $E' = E/\kappa$, since $E = V/d$ and V has decreased.
- $U' = U/\kappa$, since $U = CV^2/2$.

(b)
- $Q' = Q$, the charge remains the same, since there is nowhere for the charge to flow.
- $C' = 2C$, since we now have effectively the ~~parallel~~ series combination of two capacitors, each with 4 times the original capacitance.
- $V' = V/2$, since $V = Q/C$ and C has doubled.
- $E' = 2E$ in the two air gaps, since $E = V/d$ and V has halved while d has gone ~~done~~ down by a factor of four. E is zero in the conductive slab.
- $U' = \frac{U}{2}$, since $U = CV^2/2$

(c)

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To get the two slabs out work will need to be done, in the case of the dielectric

$$\text{work} = U - U/K = U(1 - 1/K)$$

for the metal.

$$\text{work} = U - U/2 = U/2$$

if $K > 2$ then more work needs to be done on the dielectric so less energy is converted into kinetic energy and the acceleration is less.

Answer 2

(a) Choose a gaussian surface which is a sphere, radius r, centered on the given sphere. By Gauss' Law:

$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = \frac{q_{IN}}{\epsilon_0}$$

by symmetry $|\underset{\sim}{E}|$ is constant over the Gaussian surface & $\underset{\sim}{E}$ is aligned parallel to $d\underset{\sim}{A}$

$$\therefore \oint \underset{\sim}{E} \cdot d\underset{\sim}{A} = E \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0}$$

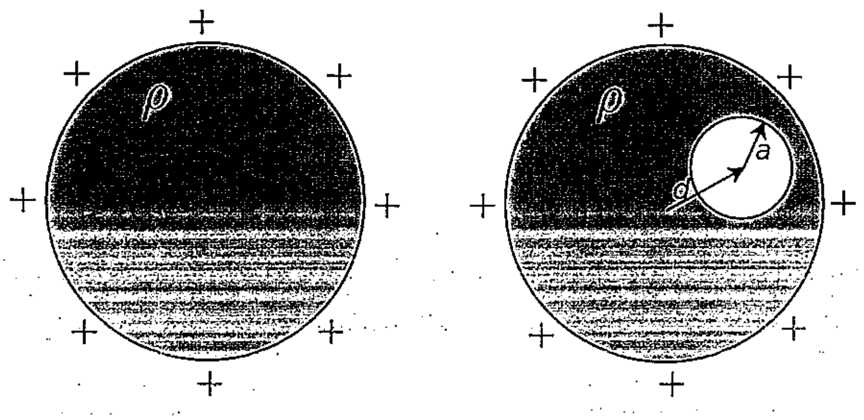
(b) Since there are no charges enclosed within the cavity, by Gauss' Law the electric field is constant within it. So, we only need to calculate E at one point. Let's use the centre of the cavity, for simplicity.

By the superimposition principle, the electric field is the sum of the original whole sphere, plus a sphere the same size/position as the cavity but with a charge density of $-\rho$. But the field in the centre of the 2nd sphere is zero, from the result of part (a), therefore the field throughout the cavity is simply E at its centre, as if the cavity was not there, i.e.,

$$E = \frac{\rho d}{3\epsilon_0}$$

directed radially outwards.

(c) The charges will be distributed on the outside of the conducting sphere:



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Answer 3

(a)

We use Ampere's Law $\oint \underset{\sim}{B} \cdot d\underset{\sim}{s} = \mu_0 I$, with an Amperian loop consisting of a circle centred on the co-axial cables, radius r.
By symmetry, $|\underset{\sim}{B}|$ is constant on the circle, & parallel to $d\underset{\sim}{s}$

$$\therefore \oint \underset{\sim}{B} \cdot d\underset{\sim}{s} = B 2\pi r = \mu_0 \times (\text{enclosed current})$$

$$\therefore B = \frac{\mu_0}{2\pi r} \times (\text{enclosed current})$$

for $r < a$ $\quad B = \frac{\mu_0}{2\pi r} I \times \frac{r^2}{a^2} = \frac{\mu_0 I r}{2\pi a^2}$

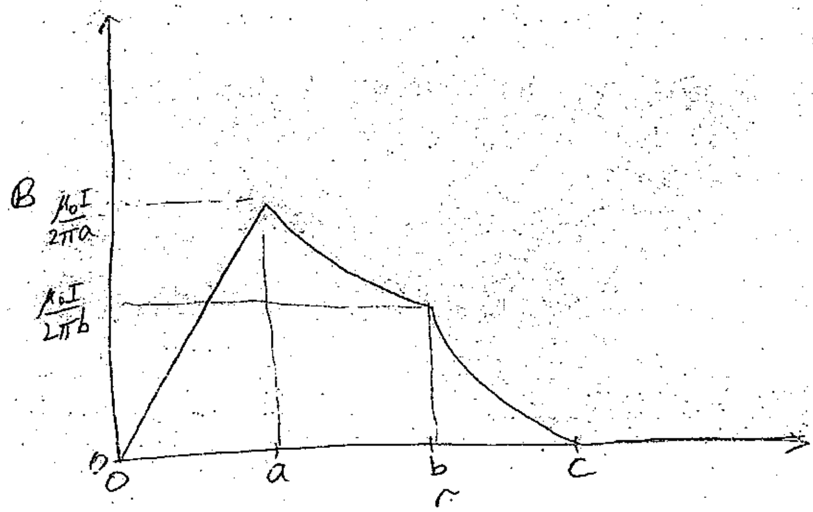
$a < r < b$ $\quad B = \frac{\mu_0 I}{2\pi r}$

$b < r < c$ $\quad B = \frac{\mu_0 I}{2\pi r}\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$

$r > c$ $\quad B = 0$

In all cases B is in the direction given by the right-hand rule, i.e. CCW as observed from above.

(b)



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(c) The electric field is zero everywhere within the two conductors, and is zero for $r > c$ (from the assumption in the question). The only question remaining is the field for $a < r < b$, which depends on the net charge on the two conductors. If we assume that this the net charge is zero (reasonable) then the electric field here is zero too.