Solutions.

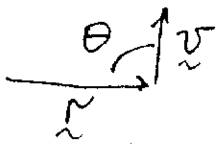
Q1. Total mark 20

(a) $v_x = \frac{dx}{dt} = -\omega r \sin\omega t$, $v_y = \frac{dy}{dt} = \omega r \cos\omega t$

(b) $\underset{\sim}{v} \cdot \underset{\sim}{r} = v_x x + v_y y = -\omega r \sin\omega t \; r\cos\omega t +$

$+ \omega r \cos\omega t \; r \sin\omega t = 0$

$\underset{\sim}{v} \cdot \underset{\sim}{r} = v r \cos\theta = 0 \qquad \cos\theta = 0 \qquad \theta = 90^\circ$



(c) $a_x = \frac{dv_x}{dt} = -\omega^2 r \cos\omega t$, $a_y = \frac{dv_y}{dt} = -\omega^2 r \sin\omega t$

$a_x = -\omega^2 x$, $a_y = -\omega^2 y$

$\underset{\sim}{a} = -\underset{\sim}{r} \cdot \omega^2$

$\underset{\sim}{a}$ is directed opposite to $\underset{\sim}{r}$

toward the centre

d). Friction force $F_{fr} \le \mu m g$

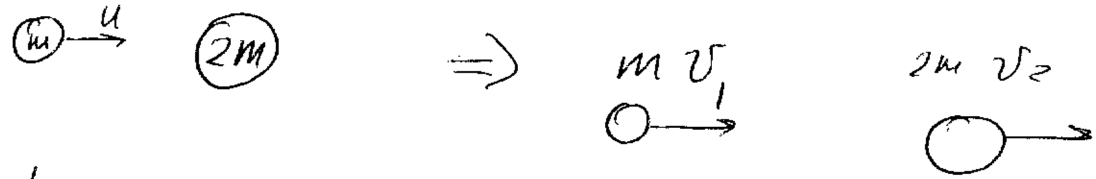
Centripetal force $\frac{mv^2}{R} = F_{fr} \le \mu m g$

$v^2 \le \mu R g$

Maximal allowed speed $v = \sqrt{\mu R g}$

Q2 (Total mark 20)

(a)



(1) Momentum $\quad mu = mv_1 + 2mv_2$

(2) Energy $\quad \frac{mu^2}{2} = \frac{m}{2}v_1^2 + \frac{2m}{2}v_2^2$

(1) $u = v_1 + 2v_2 \quad \Rightarrow 2v_2 = u - v_1$

(2) $u^2 = v_1^2 + 2v_2^2 \quad \Rightarrow 2v_2^2 = u^2 - v_1^2 = (u - v_1)(u + v_1)$

Divide $\frac{(2)}{(1)} \quad v_2 = \frac{(u - v_1)(u + v_1)}{u - v_1} = u + v_1$

add (1) $\quad 3v_2 = 2u \qquad v_2 = \frac{2}{3}u$

$v_1 = -u + v_2 = -u + \frac{2}{3}u = -\frac{1}{3}u$

$v_1 = -\frac{u}{3}$

(a) Total mark

(equations (1),(2) only -5)

(b),

initial $\quad P_{ix} = mv \sin\frac{\pi}{2} = \frac{mv}{\sqrt{2}}$

final $\quad P_{fx} = -\frac{mv}{\sqrt{2}}$

$\Delta P_x = P_{ix} - P_{fx} = \frac{mv}{\sqrt{2}} - \left(-\frac{mv}{\sqrt{2}}\right) = \sqrt{2}\,mv$

$\Delta P_y = 0$

(b) Total mark

(c). $\quad \Delta P_x = N\sqrt{2}\,mv$

$F_x = \frac{\Delta P_x}{T} = \frac{N\sqrt{2}\,mv}{T}$

(c) Total mark

Q3. Total mark 20 | Mark

(a). Centripetal force $\frac{mv^2}{R} = \frac{GMm}{R^2}$

$v_0^2 = \frac{GM}{R}$ $\quad v_0 = \sqrt{\frac{GM}{R}}$

(b) at infinity kinetic energy and potential energy is equal to zero (for the minimal escape speed).

Energy conservation $E = 0 = \frac{m_d v_e^2}{2} - \frac{G m_d m}{r} = 0$

$v_e^2 = \frac{2Gm}{r}$ $\quad v_e = \sqrt{\frac{2Gm}{r}}$

(c) The device must be sent in the same direction as that of the Earth velocity $v_0 = \sqrt{\frac{GM}{R}}$. Total velocity of device relative to the Sun is

$v_t = v_i + v_0$

To reach infinity (where $E \geq 0$) initial energy of the device must be zero or larger than zero. Minimal initial velocity corresponds to $E = 0$.

$$E = 0 = \frac{m_d (v_i + v_0)^2}{2} - \frac{G m_d m}{r} - \frac{G m_d M}{R}$$

$$(v_i + v_0)^2 = 2G\left(\frac{m}{r} + \frac{M}{R}\right)$$

$$v_i = \sqrt{2G\left(\frac{m}{r} + \frac{M}{R}\right)} - v_0 = \sqrt{2G\left(\frac{m}{r} + \frac{M}{R}\right)} - \sqrt{\frac{2GM}{r}}$$