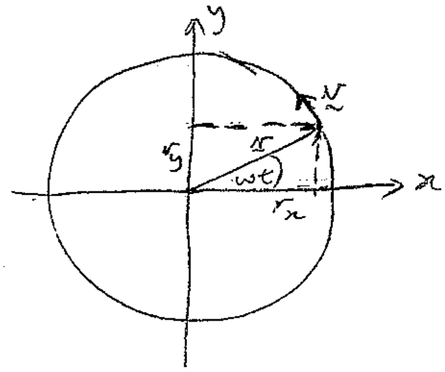
PHYS 1131 TEST 1R S2 2006

1. (a)



(i) $r_x = r\cos\omega t$

$r_y = r\sin\omega t$.

$\Rightarrow \mathbf{r} = r\cos\omega t\,\mathbf{i} + r\sin\omega t\,\mathbf{j}$

(ii) velocity $\mathbf{v} = \frac{d}{dt}\mathbf{r}$

$= \frac{d}{dt}(r\cos\omega t)\,\mathbf{i} + \frac{d}{dt}(r\sin\omega t)\,\mathbf{j}$

$= -r\omega\sin\omega t\,\mathbf{i} + r\omega\cos\omega t\,\mathbf{j}$

acceleration $\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = \frac{d\mathbf{v}}{dt}$

$= -r\omega^2\cos\omega t\,\mathbf{i} - r\omega^2\sin\omega t\,\mathbf{j}$

$= -\omega^2(r\cos\omega t\,\mathbf{i} + r\sin\omega t\,\mathbf{j})$

$\Rightarrow \mathbf{a} = -\omega^2\mathbf{r}$.

(iii) $\mathbf{v}\cdot\mathbf{r} = (-r\omega\sin\omega t\,\mathbf{i} + r\omega\cos\omega t\,\mathbf{j})\cdot(r\cos\omega t\,\mathbf{i} + r\sin\omega t\,\mathbf{j})$

$= -r^2\omega\sin\omega t.\cos\omega t + r^2\omega\cos\omega t.\sin\omega t$

$\Rightarrow \quad \underset{\sim}{v} \cdot \underset{\sim}{r} = 0$

$\Rightarrow$ $\underset{\sim}{v}$ and $\underset{\sim}{r}$ are mutually perpendicular.

(b) $\underset{\sim}{s} = \underset{\sim}{r_1} - \underset{\sim}{r} = (5.5\underset{\sim}{i} + 7.9\underset{\sim}{j} - 3.1\underset{\sim}{k})\,m$

(i) Time t_1 for stone to be displaced by $\underset{\sim}{s}$:

In time t_1, stone travels vertical distance 3.1 m

From rest,

$$s_y = -3.1 = -\tfrac{1}{2} g t^2$$

$$\Rightarrow \quad t = \sqrt{2 \times 3.1/9.8} = \overset{0.795}{0.80}\ s.$$

(ii) $\underset{\sim}{v} = \underset{\sim}{v_0} + \underset{\sim}{a} t.$

$v_x = v_{0x} \qquad \Rightarrow \quad s_x = v_{0x} t$

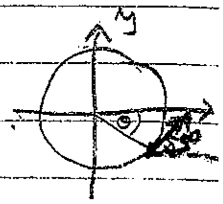
$v_y = v_{0y} \qquad \Rightarrow \quad s_y = v_{0y} t.$

$$\Rightarrow \quad v_{0x} = s_x/t = 5.5/0.80 = 6.92\ ms^{-1}$$
$$v_{0y} = s_y/t = 7.9/0.80 = 9.94\ ms^{-1}$$

$\Rightarrow$ Initial Speed $\quad v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = 12\ ms^{-1}$.

(iii) Direction of velocity when string broke:

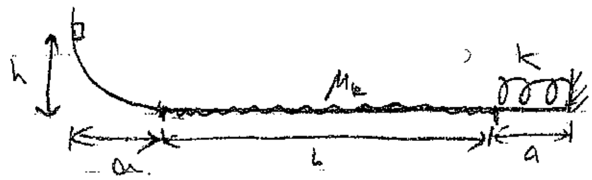
$$\theta_1 = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = 55^\circ$$



Angle of string with x-axis lags 90°

$\Rightarrow \quad \theta = -35^\circ$ or 325°

2.



(a). (i) $mgh = \frac{1}{2} m v_a^2$

$$\Rightarrow v_a = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.80 \times 0.950} = 4.32\ \mathrm{m\,s^{-1}}.$$

(ii) Mech. energy lost to friction $E_{lost} = 728\ \mathrm{mJ}$

Spring is maximally compressed when speed of block $v = 0$

$\Rightarrow$ Energy cons: $mgh - E_{lost} = \frac{1}{2} k x^2$.

$$\Rightarrow x = \sqrt{\frac{2}{k}\left(mgh - E_{lost}\right)}$$

$$k = 4.36\ \mathrm{N/cm} = 436\ \mathrm{N/m}.$$

$$\Rightarrow x = \sqrt{\frac{2}{436}\left(0.254 \times 9.80 \times 0.950 - 728 \times 10^{-3}\right)}$$

$$= 0.0866\ \mathrm{m} = 8.66\ \mathrm{cm}.$$

(iii) On return to incline, $2 \times 728\ \mathrm{mJ}$ is lost

$$\Rightarrow mgh - 2 \times 0.728 = mgh'$$

$$\Rightarrow h' = h - \frac{1}{mg}(2 \times 0.728)$$

$$= 0.950 - \frac{1}{0.254 \times 9.80}(2 \times 0.728)$$

$$= 0.365\ \mathrm{m}.$$

(b) The block comes to rest on rough surface when all mechanical energy is lost due to friction ie, when $E_{lost} = mgh$.

On one trip across rough surface, 0.728 J is lost.

$$\Rightarrow \quad mgh = 0.254 \times 9.80 \times 0.950$$

$$= 2.36 \text{ J}.$$

$$\Rightarrow \quad 2.36 / 0.728 = 3.24$$

⇒ block traversed rough surface 3.24 times before coming to rest

⇒ block is a fraction 0.24 of the length of the rough surface from spring when at rest, ie

$$x = a + 0.76L$$
$$= 0.35 + 0.76 \times 2.37 = 2.15 \text{ m}$$

(c). Coefficient of kinetic friction:

Energy lost due to friction on travelling length L over rough surface

$$E_{lost} = 0.728 = f_k L = \mu_k N L = \mu_k mgL$$

$$\Rightarrow \mu_k = 0.728 / (mgL)$$
$$= 0.728 / (0.254 \times 9.80 \times 2.37)$$
$$= 0.123.$$

(d). If $\mu_k' = \frac{1}{2}\mu_k$, block would travel twice distance on rough surface before stopping.

$\Rightarrow \quad 2.36/(7.28/2) = 6.48$ lengths travelled.

$\Rightarrow \quad x = a + 0.48L = 0.35 + 0.48 \times 2.37$

$= 1.49 \text{ m}$

3. (a). $m = 2.0 \times 10^{30}$ kg
$R = 2.2 \times 10^{20}$ m.
$T = 2.5 \times 10^{8}$ years $= 7.9 \times 10^{15}$ s.

(i) $v = \frac{2\pi R}{T} = \frac{2\pi \times 2.2 \times 10^{20}}{7.9 \times 10^{15}} = 1.7 \times 10^{5}$ ms^{-1}.

(ii). $mv^2/R = \frac{GmM}{R^2}$.

(iii) Mass of galaxy: $M = v^2 R / G$.

$= (1.75 \times 10^{5})^2 \times 2.2 \times 10^{20} / 1.67 \times 10^{-11}$

$= 4.0 \times 10^{41}$ kg.

Assume mass of galaxy comprised entirely of stars.

Take mass of Sun to be typical mass of star.

$\Rightarrow$ Number of stars in Milky Way $\simeq 4.0 \times 10^{41} / 2.0 \times 10^{30}$
$= 2 \times 10^{11}$.

(b). Mass inside radius R_2: $M_2 = v_2^2 R_2 / G$
Mass inside radius R_1: $M_1 = v_1^2 R_1 / G$.

Mass inside spherical shell $R_2 - R_1$:
$$M_2 - M_1 = \frac{v_2^2 R_2}{G} - \frac{v_1^2 R_1}{G}.$$

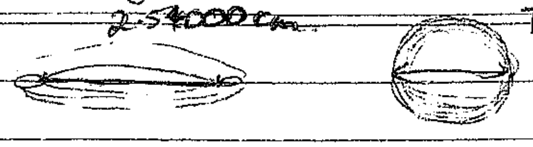
(c) If no dark matter, $M_2 - M_1 = 0 = \frac{v_2^2 R_2}{G} - \frac{v_1^2 R_1}{G}$.
$\Rightarrow v_2^2 R_2 = v_1^2 R_1 \Rightarrow v_2 / v_1 = \sqrt{R_1/R_2}$

$m_c = 104\,g = 0.104\,kg$

$T_c = 0°C$

2.54000 cm

4.(a)



$T_a = 100°C$

2.54533 cm

$$-m_a c_a \Delta T_a = m_c c_c \Delta T_c$$

$c_c = 387\,J/(kg\,K)$

$c_a = 900\,J/(kg \cdot K)$

$$-m_a c_a (T - T_a) = m_c c_c (T - T_c)$$

$$m_a = -\frac{m_c c_c (T - T_c)}{c_a (T - T_a)}$$

c_c, c_a - given

need to find equilibrium temperature T.

$$\Delta L_a = \alpha_a L_a \Delta T_a \Rightarrow L - L_a = \alpha_a L_a (T - T_a)$$

$$\Delta L_c = \alpha_c L_c \Delta T_c \Rightarrow L - L_c = \alpha_c L_c (T - T_c)$$

$\Rightarrow$ Final diameters same.

$$\Rightarrow \alpha_a L_a T - \alpha_a L_a T_a + L_a = \alpha_c L_c T - \alpha_c L_c T_c + L_c$$

$$\Rightarrow T(\alpha_a L_a - \alpha_c L_c) = L_a(\alpha_a T_a - 1) - L_c(\alpha_c T_c - 1)$$

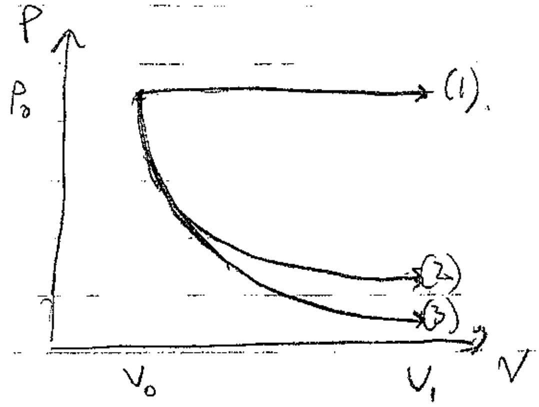
$$\Rightarrow T = \frac{L_a(\alpha_a T_a - 1) - L_c(\alpha_c T_c - 1)}{\alpha_a L_a - \alpha_a L_c}$$

$$\Rightarrow T = \left[2.54533 \times 10^{-2}\left(23 \times 10^{-6} \times 373 - 1\right) - 2.54000 \times 10^{-2}\left(17 \times 10^{-6} \times 273 - 1\right)\right] / \left[23 \times 10^{-6} \times 2.54533 \times 10^{-2} - 17 \times 10^{-6} \times 2.54000 \times 10^{-2}\right]$$

$$= 307.1\,K = 34.0°C$$

$$\Rightarrow m_a = -\frac{0.104 \times 387 \times (34 - 0)}{900 \times (34 - 100)} = 23\,g$$

(b) (i)



(ii) Work done on gas is greatest in (3) since work done is negative and given by area under curve $-\int P\,dV$. Path (3) gives smallest negative work done.

Least amount of work done on Gas is given by (1) since $-\int P\,dV$ is the largest (absolute value) and it is negative.

(iii) Heat added: $\Delta E_{int} = Q + W$
$\Rightarrow Q = \Delta E_{int} - W$.

In all cases, W is negative, so $-W$ is positive.

For (1), $-W$ is the largest and so is ΔE_{int} (since (1) corresponds to largest ΔT).
$\Rightarrow$ Path (1) gives greatest heat added.

Least heat added is from Path (3), since here $-W$ is the smallest and ΔE_{int} is negative.

(iv) ΔE_{int} is greatest in (1) since ΔT is largest.
ΔE_{int} is least in (3) since ΔT is negative.

($\Delta E_{int} = 0$ for (2))