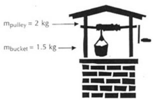
QUESTION 4 [Marks 14]

A frictionless winding pulley at the top of a well is in the shape of a cylinder. The mass of the winding pulley is $m_{pulley} = 2$ kg and its radius is $r_{pulley} = 0.15$ m.



There is a light rope wrapped round the pulley which is vertically connected to a bucket of mass $m_{bucket} = 1.5$ kg so the bucket is free to drop down into a well.

Initially the system is at rest and the bucket then drops down into the well.

The moment of inertia of a cylinder of mass M and radius R is $I = \frac{1}{2} MR^2$.

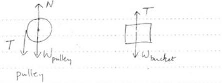
(a) Draw a sketch showing on the forces acting on the bucket and all the forces acting on the pulley.

(b) (i) Write down an expression for the acceleration ***a*** of the bucket.
(ii) Write down an expression for the angular acceleration α of the pulley.
(iii) Relate the acceleration **a** to the angular acceleration α.

(c) (i) Determine the tension T in the rope.
(ii) Determine the acceleration ***a*** of the bucket.
(iii) Determine the angular acceleration α of the pulley.

(d) Determine the torque τ acting on the pulley about the axis of rotation.

(a)



(b) i. $m_{bucket} a = W_{bucket} - T \quad \therefore a = \frac{W_{bucket} - T}{m_{bucket}} = g - \frac{T}{m_{bucket}}$

using $F_{net} = ma$

(ii) use $\tau_{net} = I\alpha$

$\tau_{net} = T r_{pulley} = T \times 0.15 = I\alpha$

$\alpha = \frac{T\, 0.15}{I}$ where $I = \frac{1}{2} m_{pulley} r_{pulley}^2$

$= T/0.15$ $\qquad = \frac{1}{2}\, 2 \times (0.15)^2$