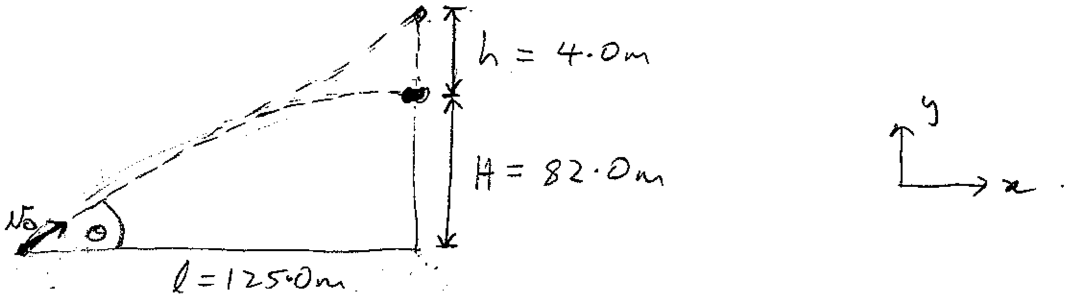
1 (a).



(i) In time t, target falls a distance h:

$$h = \tfrac{1}{2} g t^2 \qquad \text{(falls from rest)}.$$

$$\Rightarrow t = \sqrt{2h/g} = \sqrt{2\times 4.0/9.8} = 0.904\ s.$$

(ii) Initial speed of cannon-ball $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$

$$l = v_{0x} t$$
$$H = v_{0y} t - \tfrac{1}{2} g t^2.$$

Need time t (from (i)), time between firing of cannon-ball and collision with target.

$$\Rightarrow \quad v_{0x} = l/t = 125/0.904 = 138.3\ ms^{-1}.$$

$$v_{0y} = \tfrac{1}{t}\left(H + \tfrac{1}{2} g t^2\right) = \tfrac{1}{0.904}\left(82.0 + \tfrac{1}{2}\,9.8 \times (0.904)^2\right)$$
$$= 95.14\ ms^{-1}.$$

$$\Rightarrow v_0 = \sqrt{(138.3)^2 + (95.14)^2} = 168\ ms^{-1}.$$

(iii). For cannon to hit target, cannon must have been aimed at target (both cannon-ball and target fall same vertical distance)

So $\tan\theta = h+H/l$. $\Rightarrow \theta = \tan^{-1}(h+H/l)$
$$= \tan^{-1}(86.0/125) = 34.5°.$$

b) (i) The largest distance the car could travel is L.
(Carriage will shift each time a cannon-ball is fired s.t. the centre of mass of the system stays at the same point. The car will travel a distance L when the total mass of the cannon-balls is much greater than the mass of the carriage)

(ii) After all cannon-balls have reached the wall, the speed of the car is zero.
(Since the centre of mass is at rest and the cannon-balls are at rest, the car must also be at rest.)

2(a) $m_1 = 2m$

$m_2 = m$

Centre of mass $(m_1 + m_2)\mathbf{r}_{cm} = m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2$.

In centre of mass frame, $\mathbf{r}_{cm} = 0$

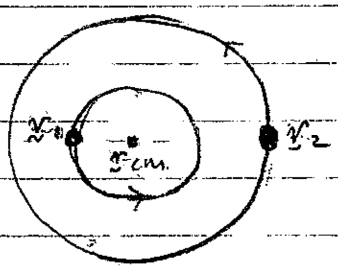
$$\Rightarrow \quad 0 = m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2$$

$$\Rightarrow \quad 0 = 2\cancel{m}\,\mathbf{r}_1 + \cancel{m}\,\mathbf{r}_2.$$

$$\Rightarrow \quad \mathbf{r}_1 = -\mathbf{r}_2/2.$$

Trajectories:

In general, the particles move in elliptical orbits. Shown case for circular motion



(b) centre of mass velocity: $\mathbf{v}_{cm} = \frac{1}{m_1 + m_2}\left(m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2\right)$

(c) $$K = \tfrac{1}{2}\mu v^2 + \tfrac{1}{2}(m_1 + m_2) v_{cm}^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\mathbf{v}_1 - \mathbf{v}_2)^2 + \frac{1}{2}(m_1 + m_2)\mathbf{v}_{cm}^2$$

$$\Rightarrow K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(v_1^2 + v_2^2 - 2\mathbf{v}_1 \cdot \mathbf{v}_2\right) + \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2}\right)^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(v_1^2 + v_2^2 - 2\mathbf{v}_1 \cdot \mathbf{v}_2\right) + \frac{1}{2}\frac{\left(m_1^2 v_1^2 + m_2^2 v_2^2 + 2 m_1 m_2 \mathbf{v}_1 \cdot \mathbf{v}_2\right)}{m_1 + m_2}$$

$$\Rightarrow K = \frac{1}{2}\,\frac{m_1 m_2}{m_1+m_2}\left(v_1^2+v_2^2-\cancel{2\underset{\sim}{v_1}\cdot\underset{\sim}{v_2}}\right) + \cancel{\frac{m_1 m_2}{m_1+m_2}\,\underset{\sim}{v_1}\cdot\underset{\sim}{v_2}} + \frac{1}{2}\,\frac{1}{m_1+m_2}\left(m_1^2v_1^2+m_2^2v_2^2\right)$$

$$= \frac{1}{2}\,\frac{1}{m_1+m_2}\,v_1^2\left(m_1m_2+m_1^2\right) + \frac{1}{2}\,\frac{1}{m_1+m_2}\,v_2^2\left(m_1m_2+m_2^2\right)$$

$$= \frac{1}{2}\,\cancel{\frac{1}{m_1+m_2}}\,v_1^2\,m_1\cancel{\left(m_2+m_1\right)} + \frac{1}{2}\,\cancel{\frac{1}{m_1+m_2}}\,v_2^2\,m_2\cancel{\left(m_1+m_2\right)}$$

$$\Rightarrow K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 .$$

This is kinetic energy of two particles.

(d) First term is kinetic energy of two particles in centre of mass reference frame. Second term corresponds to motion of whole system (motion of centre of mass).

(e) Change in potential energy

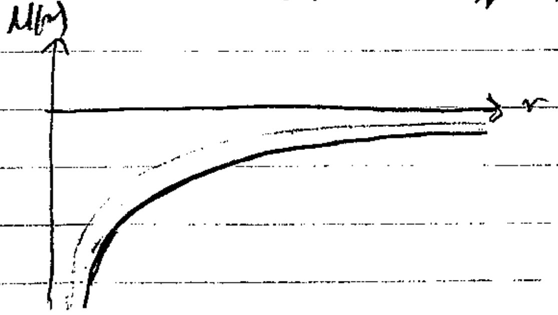
$$\Delta U = U(r) - U(r=\infty) = -\int_{\infty}^{r}\underset{\sim}{F}\cdot d\underset{\sim}{s} = +\int_{\infty}^{r}\left(+\frac{kq_1q_2}{r^2}\right)dr$$

$$\Rightarrow U(r) = kq_1q_2\int_{\infty}^{r}\frac{1}{r^2}\,dr + \underbrace{U(r=\infty)}_{=0}$$

$$= kq_1q_2\left\{-\frac{1}{r}\right\}_{\infty}^{r}$$

$$= kq_1q_2\left(-\frac{1}{r}+\underbrace{\frac{1}{\infty}}_{=0}\right).$$

$$\Rightarrow U(r) = -\frac{kq_1q_2}{r}$$



f). Minimum energy required to separate particles to infinity is energy to give total mechanical energy $E = 0$

$\Rightarrow$ minimum energy required

$$K + U + E_{req} = 0$$

$$\Rightarrow E_{req} = -U - K = \frac{kq_1q_2}{r} - \frac{1}{2}\mu v^2 .$$

3.(a). Momentum conservation:

$$m\underset{\sim}{v} = m\underset{\sim}{v}/2 + M\underset{\sim}{V}.$$

All move in one direction (immediately before and after collision).

$$\Rightarrow \quad mv = mv/2 + MV.$$
$$\Rightarrow \quad MV = mv/2.$$
$$\Rightarrow \quad V = \frac{m}{M} \cdot v/2.$$

(b). Mechanical energy lost is equal to change in kinetic energy

$$E_{lost} = \Delta K = K_f - K_i$$
$$= \frac{1}{2} m \left(\frac{v}{2}\right)^2 + \frac{1}{2} MV^2 - \frac{1}{2} mv^2$$
$$= \frac{1}{8} mv^2 + \frac{1}{2} M \cdot \left(\frac{m}{M} \frac{v}{2}\right)^2 - \frac{1}{2} mv^2.$$
$$= -\frac{3}{8} mv^2 + \frac{1}{8} \frac{m^2}{M} v^2.$$
$$= \frac{1}{8} mv^2 \left(-3 + \frac{m}{M}\right)$$

(c). No, loss of mechanical energy does not violate law of conservation of energy.

Examples of where this energy goes (one is enough):
- internal energy of bob / bullet through friction (temperature increases).
- sound

- - - - -

(d). Speed of bob at A: $\frac{1}{2}\cancel{M}V_A^2 + \cancel{M}g(2l) = \frac{1}{2}\cancel{M}V^2$

$$\frac{1}{2}V_A^2 + 2gl = \frac{1}{2}\cdot\left(\frac{m}{M}\cdot\frac{v}{2}\right)^2 = \frac{1}{8}\frac{m}{M}v^2$$

$$\Rightarrow V_A = \sqrt{\frac{1}{4}\left(\frac{m}{M}v\right)^2 - 4gl}$$

(e).

T – tension in wire

(f) Bob will not reach A if $T = 0$

Bob will swing a complete circle if $T > 0$.

$mg + T = ma = mV_A^2/l$ Newton's 2nd law

$\Rightarrow T = mV_A^2/l - mg > 0$

(g). $\Rightarrow V_A^2 > gl$

minimum $V_A^2 = gl$.

$\Rightarrow$ minimum speed v:

$$V_A^2 = \frac{1}{4}\left(\frac{m}{M}\right)^2 v^2 - 4gl = gl.$$

$$\Rightarrow \frac{1}{4}\left(\frac{m}{M}\right)^2 v^2 = 5gl$$

$$\Rightarrow \text{minimum } v = 2\frac{M}{m}\sqrt{5gl}$$

4. (a). $\Delta T = 3000 - 300 = 2700\ K$
$\beta = 3.2 \times 10^{-5}\ K^{-1}$.

Want to find change in radius $\Delta R / R$.

$\Delta V = \beta V \Delta T$

Original volume $V = \frac{4}{3} \pi R^3$.

Change in volume $\Delta V = \frac{4}{3}\pi \left[(R + \Delta R)^3 - R^3 \right]$

$$= \frac{4}{3} \pi R^3 . \left[\left(1 + \frac{\Delta R}{R}\right)^3 - 1 \right].$$

$$= \frac{4}{3} \pi R^3 . \left[\cancel{1} + \frac{3\Delta R}{R} + \dots - \cancel{1} \right]$$

$$\simeq \underbrace{\frac{4}{3} \pi R^3}_{V} . \frac{3\Delta R}{R}$$

Can write from beginning if remember that $\beta \approx 3\alpha$, $\Rightarrow \alpha \simeq \beta/3$.

$\Rightarrow \Delta R = \alpha R \Delta T$

$\simeq \beta/3 . R \Delta T$.

$\Rightarrow \Delta R / R = \beta/3 . \Delta T$

$$\Rightarrow \Delta V = V . \frac{3\Delta R}{R}.$$

$$\Rightarrow \Delta V / V = \frac{3 \Delta R}{R} = \beta \Delta T.$$

$$\Rightarrow \Delta R / R = \beta/3 . \Delta T.$$

$$= 3.2 \times 10^{-5} / 3 . (2700).$$

$$= 2.9 \times 10^{-2}.$$

$\Rightarrow$ radius has increased by 2.9 %.

(b) Atmospheric pressure $P = 1.01 \times 10^5$ Pa

Pressure $P = F/A$, $F = mg$

Surface area ~~of~~ Earth: $A = 4\pi R_E^2$

$= 4\pi \times (6.37 \times 10^6)^2$

$= 5.10 \times 10^{14}\ m^2$

$\Rightarrow \quad P = mg/A$

$\Rightarrow m = PA/g = 1.01 \times 10^5 \times 5.10 \times 10^{14} / 9.80$

$= 5.25 \times 10^{18}$

Mass of Earth $M_E = 5.98 \times 10^{24}$ kg

$\Rightarrow$ mass of atmosphere: $m = 8.8 \times 10^{-7} M_E$

(c) (i). First law of thermodynamics:

$$\underbrace{\Delta E_{int}}_{\text{change in internal energy of system.}} = \underbrace{Q}_{\text{heat added to system}} + W \leftarrow \text{work done on system}.$$

The change in internal energy of a thermodynamic process is equal to the sum of the heat added to the system and the work done on the system.

(statement of energy conservation).

(ii). In one cycle, internal energy change is zero. (comes back to same point corresponding to one particular temperature).

(iii) From 1st law of thermodynamics

$$\Delta E_{int} = Q + W$$

In one complete cycle, change in internal energy is zero

$$\Rightarrow \quad 0 = Q + W.$$

$$\Rightarrow \quad \underline{Q = -W}.$$

Work done to system is negative of area of loop.

Area of each square $10\times10^6 \times 1\times10^{-3} = 10^4$ J

Area of half-circle: $\frac{1}{2}\pi R^2$

Area of rectangle (side R by 2R): $2R^2$.

$\Rightarrow$ Work added to system $W = \left(\frac{1}{2}\pi \cancel{R^2} / 2\cancel{R^2}\right) \times 4.5\times10^4$

$$= \underline{3.5\times10^4 \text{ J}}.$$

(Work is positive because path is anti-clockwise, ie, more work associated with compression than expansion)

$\Rightarrow$ net heat added to system in one cycle is 5

$$\underline{Q = -3.5\times10^4 \text{ J}}.$$

ie, 3.5×10^4 J of energy is lost by heat