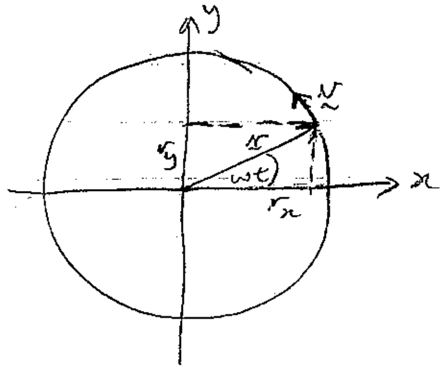
1. (a)



(i) $r_x = r\cos\omega t$

$r_y = r\sin\omega t$.

$\Rightarrow \underset{\sim}{r} = r\cos\omega t\ \hat{i} + r\sin\omega t\ \hat{j}$

(ii) velocity $\underset{\sim}{v} = \frac{d}{dt}\underset{\sim}{r}$

$= \frac{d}{dt}(r\cos\omega t)\underset{\sim}{i} + \frac{d}{dt}(r\sin\omega t)\underset{\sim}{j}$

$= -r\omega\sin\omega t\ \underset{\sim}{i} + r\omega\cos\omega t\ \underset{\sim}{j}$

acceleration $\underset{\sim}{a} = \frac{d^2 \underset{\sim}{r}}{dt^2} = \frac{d\underset{\sim}{v}}{dt}$

$= -r\omega^2\cos\omega t\ \underset{\sim}{i} - r\omega^2\sin\omega t\ \underset{\sim}{j}$

$= -\omega^2(r\cos\omega t\ \underset{\sim}{i} + r\sin\omega t\ \underset{\sim}{j})$

$\Rightarrow \underset{\sim}{a} = -\omega^2\underset{\sim}{r}$.

(iii) $\underset{\sim}{v}\cdot\underset{\sim}{r} = (-r\omega\sin\omega t\ \underset{\sim}{i} + r\omega\cos\omega t\ \underset{\sim}{j})\cdot(r\cos\omega t\ \underset{\sim}{i} + r\sin\omega t\ \underset{\sim}{j})$

$= -r^2\omega\sin\omega t.\cos\omega t + r^2\omega\cos\omega t.\sin\omega t$

$\Rightarrow \quad \underset{\sim}{v} \cdot \underset{\sim}{r} = 0$

$\Rightarrow$ $\underset{\sim}{v}$ and $\underset{\sim}{r}$ are mutually perpendicular.

(b) $\underset{\sim}{s} = \underset{\sim}{r}_1 - \underset{\sim}{r} = (5.5\,\underset{\sim}{i} + 7.9\,\underset{\sim}{j} - 3.1\,\underset{\sim}{k})\,m$

(i) Time t_1 for stone to be displaced by $\underset{\sim}{s}$:

In time t_1, stone travels vertical distance 3.1 m from rest,

$$s_y = -3.1 = -\tfrac{1}{2} g t^2$$

$$\Rightarrow \quad t = \sqrt{2 \times 3.1 / 9.8} = \overset{0.795}{0.80\ s.}$$

(ii) $\underset{\sim}{v} = \underset{\sim}{v}_0 + \underset{\sim}{a}\,t$.

$v_x = v_{0x} \quad \Rightarrow \quad s_x = v_{0x} t$

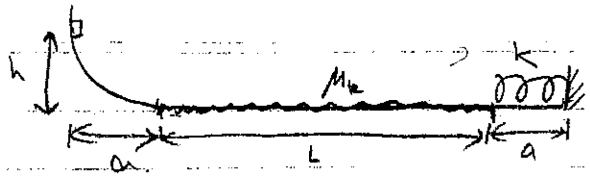
$v_y = v_{0y} \quad \Rightarrow \quad s_y = v_{0y} t$

$\Rightarrow \quad v_{0x} = s_x / t = 5.5/0.80 = 6.92\ ms^{-1}$

$ v_{0y} = s_y / t = 7.9/0.80 = 9.94\ ms^{-1}$

$\Rightarrow$ Initial Speed $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = 12\ ms^{-1}$.

2.



(a). (i) $mgh = \frac{1}{2} m v_a^2$

$\Rightarrow v_a = \sqrt{2gh}$

$= \sqrt{2 \times 9.80 \times 0.950} = 4.32 \text{ ms}^{-1}$.

(ii) Mech. energy lost to ~~friction~~ $E_{lost} = 728$ mJ

Spring is maximally compressed when speed of block $v=0$

$\Rightarrow$ Energy cons: $mgh - E_{lost} = \frac{1}{2}kx^2$

$\Rightarrow x = \sqrt{\frac{2}{k}(mgh - E_{lost})}$

$\Rightarrow k = 4.36$ N/cm $= 436$ N/m

$\Rightarrow x = \sqrt{\frac{2}{436}(0.254 \times 9.80 \times 0.950 - 728 \times 10^{-3})}$

$= 0.0866$ m $= 8.66$ cm

(iii) On return to incline, 2×728 mJ is lost

$\Rightarrow mgh - 2 \times 0.728 = mgh'$

$\Rightarrow h' = h - \frac{1}{mg}(2 \times 0.728)$

$= 0.950 - \frac{1}{0.254 \times 9.80}(2 \times 0.728)$

$= 0.365$ m

(b) The block comes to rest on rough surface when all mechanical energy is lost due to friction, ie, when $E_{lost} = mgh$.

On one trip across rough surface, 0.728 J is lost.

$$\Rightarrow \quad mgh = 0.254 \times 9.80 \times 0.950$$

$$= 2.36 \text{ J}$$

$$\Rightarrow \quad 2.36 / 0.728 = 3.24$$

$\Rightarrow$ block traversed rough surface 3.24 times before coming to rest

$\Rightarrow$ block is a fraction 0.24 of the length of the rough surface from spring when at rest, ie

$$x = a + 0.76L$$

$$= 0.35 + 0.76 \times 2.37 = 2.15 \text{ m}$$

3. $m = 2.0 \times 10^{30}$ kg

$R = 2.2 \times 10^{20}$ m.

$T = 2.5 \times 10^{8}$ years $= 7.9 \times 10^{15}$ s.

(a) $v = \frac{2\pi R}{T} = \frac{2\pi \times 2.2 \times 10^{20}}{7.9 \times 10^{15}} = 1.7 \times 10^{5}\ ms^{-1}$.

(b) $mv^2/R = \frac{GmM}{R^2}$

(c) Mass of galaxy: $M = v^2 R / G$.

$= (1.75 \times 10^{5})^2 \times 2.2 \times 10^{20} / 6.67 \times 10^{-11}$

$= 4.0 \times 10^{41}$ kg.

Assume mass of galaxy comprised entirely of stars.

Take mass of Sun to be typical mass of star.

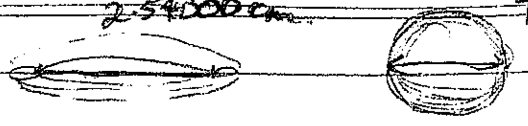
$\Rightarrow$ Number of stars in Milky Way $\simeq 4.0 \times 10^{41} / 2.0 \times 10^{30}$

$= 2 \times 10^{11}$

$m_c = 104 g = 0.104 kg$

$T_c = 0°C$

~~2.54000 cm~~ ~~$T_a = 100°C$~~

4.(a)  2.54533 cm

(i) need to find equilibrium temperature T.

$$\Delta L_a = \alpha_a L_a \Delta T_a \Rightarrow L - L_a = \alpha_a L_a (T - T_a)$$

$$\Delta L_c = \alpha_c L_c \Delta T_c \Rightarrow L - L_c = \alpha_c L_c (T - T_c)$$

⇒ Final diameters same.

$$\Rightarrow \alpha_a L_a T - \alpha_a L_a T_a + L_a = \alpha_c L_c T - \alpha_c L_c T_c + L_c$$

$$\Rightarrow T(\alpha_a L_a - \alpha_c L_c) = L_a(\alpha_a T_a - 1) - L_c(\alpha_c T_c - 1)$$

$$\Rightarrow T = \frac{L_a(\alpha_a T_a - 1) - L_c(\alpha_c T_c - 1)}{\alpha_a L_a - \alpha_a L_c}$$

$$\Rightarrow T = \left[2.54533\times10^{-2}(23\times10^{-6}\times373 - 1) - 2.54000\times10^{-2}(17\times10^{-6}\times273 - 1)\right] \times \left[23\times10^{-6}\times2.54533\times10^{-2} - 17\times10^{-6}\times2.54000\times10^{-2}\right]^{-1}$$

$$= 307.1 K = 34.0°C.$$

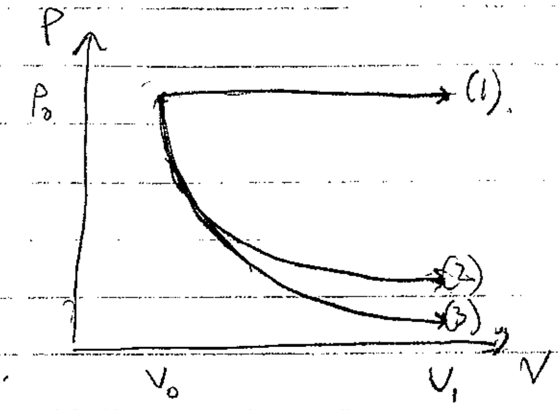
(ii) $-m_a c_a \Delta T_a = m_c c_c \Delta T_c$ $\qquad c_c = 387 J/(kg K)$, $c_a = 900 J/(kg.K)$

$$-m_a c_a (T - T_a) = m_c c_c (T - T_c)$$

$$m_a = -\frac{m_c c_c (T - T_c)}{c_a (T - T_a)}$$ $\qquad c_c, c_a$ - given

$$\Rightarrow m_a = -\frac{0.104 \times 387 \times (34 - 0)}{900 \times (34 - 100)} = 23 g$$

(b) (i)



(ii) Work done on gas is greatest in (3) since work done is negative and given by area under curve $-\int P dV$. Path (3) gives smallest negative work done.

Least amount of work done on gas is given by (1) since $-\int P dV$ is the largest (absolute value) and it is negative.

(iii) Heat added: $\Delta E_{int} = Q + W$

$\Rightarrow Q = \Delta E_{int} - W$,

In all cases, W is negative, so $-W$ is positive.

For (1), $-W$ is the largest and so is ΔE_{int} (since (1) corresponds to largest ΔT).
$\Rightarrow$ Path (1) gives greatest heat added.

Least heat added is from Path (3), since here $-W$ is the smallest and ΔE_{int} is negative.

(iv) ΔE_{int} is greatest in (1) since ΔT is largest.
ΔE_{int} is least in (3) since ΔT is negative.

($\Delta E_{int} = 0$ for (2))